PERIOD–V

M12CH18

TOPIC 18

Plane Geometry

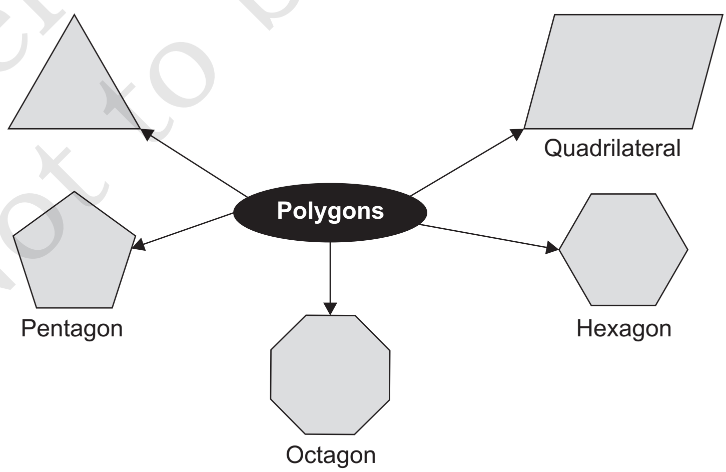
18.1. WHAT ARE POLYGONS?

A polygon is a closed figure made up of line segments (not curves) in a two-dimensionsal plane. Polygon is the combination of two words *i.e.,* poly (means many) and gon (means sides)

A minimum of three line segments is required to connect end to end, to make a closed figure. Thus a polygon with a minimum of three sides is known as Triangle and it is also called 3-gon. An *n*-sided polygon is called *n*-gon.

18.2. POLYGON SHAPE

By definition we know that the polygon is made up of line segments. Below are the shapes of some polygons that are enclosed by the different number of line segments.



18.3. TYPES OF POLYGON

Depending on the sides and angles, the polygons are classified into different types, namely

- Regular Polygon
- Irregular Polygon
- Convex Polygon
- Concave Polygon

Regular Polygon

If all the sides and interior angles of the polygon are equal, then it is known as a regular polygon.

The examples of regular polygons are square, rhombus, equilateral triangle etc.

Irregular Polygon

If all the sides and the interior angles of the polygon are of different measure, then it is known as an irregular polygon.

For example, a scalene triangle, a rectangle, a kite, etc.

Convex Polygon

If all the interior angles of a polygon are strictly less than 180 degrees, then it is known as an Convex polygon. The vertex will point outwards from the centre of the shape.

Concave Polygon

If one or more interior angles of a polygon are more than 180 degrees, then it is known as a concave polygon. A concave polygon can have atleast four sides. The vertex points towards the inside of the polygon.

However, a number of polygons are defined based on the number of sides, angles and properties.

18.4. ANGLES OF POLYGON

As we know any polygon has many vertices as it has sides. Each corner has a certain measures of angles. These angles are categorized into two types namely interior angles and exterior angles of a polygon.

18.5. INTERIOR ANGLE PROPERTY

The sum of all the interior angles of a simple n.gon = $(n - 2) \times 180$.

or

Sum = $(n - 2)n$ radius

where 'n' is equal to the number of sides of a polygon

For example, a quadrilateral has four sides, therefore, the sum of the all the interior angles is given by

Sum of interior angles of 4-sided polygon

$$= (4 - 2) \times 180° = 2 \times 180° = 360°$$

18.6. EXTERIOR ANGLE PROPERTY

The sum of interior and the corresponding exterior angles at each vertex of any polygon are supplementary to each other. For a polygon.

- Interior angle + Exterior angle = 180 degrees
- Exterior angle = 180 degrees – Interior angle

Properties

The properties of polygons are based on their sides and angles:

- The sum of all the interior angles of an n-sided polygon is $(n - 2) \times 180°$
- The number of diagonals in a polygon with n sides $= n(n - 3)/2$
- The number of triangles formed by joining the diagonals from one corner of a polygon = $n - 2$
- The measure of each interior angle of n-sided regular polygon is $[(n - 2) \times 180°]/n$
- The measure of each exterior angle of an n-sided regular polygon $= 360°/n$

18.7. AREA AND PERIMETER FORMULAS

The area and perimeter of different polygons are based on the sides.

Area: Area is defined as the region covered by a polygon in a two-dimensional plane.

Perimeter: Perimeter of a polygon is the total distance covered by the sides of a polygon.

The formulas of area and perimeter for different polygons are given below:

Name of polygon	***Area***	***Perimeter***
Triangle	½ × (base) × (height)	$a + b + c$
Square	side 2	4(side)
Rectangle	Length × Breadth	2(Length + Breadth)
Parallelogram	Base × Height	2(Sum of pair of adjacent sides)
Trapezoid	Area = 1/2(sum of parallel side)height	Sum of all sides
Rhombus	½(Product of diagonals)	4 × sides
Pentagon	$\frac{1}{4}\sqrt{5(5+2\sqrt{5})}$ side^2	Sum of all five sides
Hexagon	$3\sqrt{3}/2$ (side)^2	Sum of all six sides

Let us see one of the frequently used and the primary types of polygon, *i.e.,* triangle.

18.8. TRIANGLES (3-GON)

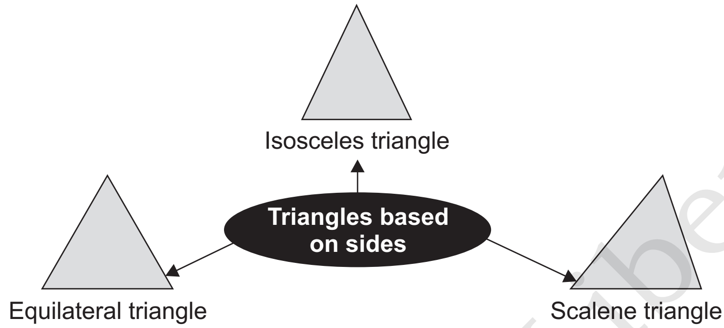
A triangle is the simplest form of the polygon that has three sides and three vertices. The triangles are also classified into different types, ababased on the sides and angles.

The sum of all the angles of the triangles is always equal to 180°(Straight angle).

18.9. TRIANGLES – BASED ON SIDES

- **Equilateral triangle:** Having all sides equal and angles of equal measure it is also called an equiangular triangle.
- **Isosceles triangle:** Having any 2 sides equal and angles opposite to the equal sides are equal
- **Scalene triangle:** Has all the 3 sides unequal.

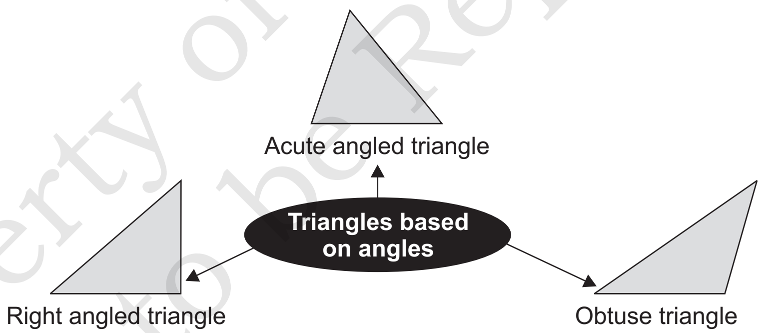
See the below figure to see the difference between the three types of triangles.



18.10. TRIANGLES – BASED ON ANGLES

- **Acute angled triangle –** Each angle is less than 90°.
- **Right angled triangle –** Any one of the three angles equal to 90°.
- **Obtuse angled triangle –** Any one angle is greater than 90°.

The below figure shows the three types of angles, based on angles.



Example 1. *A polygon is an octagon and its side length is 6 cm. Calculate its perimeter and value of one interior angle.*

Solution. The polygon is an octagon, so we have, $n = 8$

Length on one sides, $s = 6$ cm

The perimeter of the octagon

$$P = n \times s$$

$$P = 8 \times 6 = 48 \text{ cm}$$

Now, for the interior angle, we have

Interior angle of a regula polygon $= \dfrac{(n-2)180}{n}$

$$= (8 - 2) \times 180/8$$
$$= 6 \times 180/8 = 135°$$

Therefore, the perimeter of the octagon is 48 cm and the value of one of the interior angles is 135°.

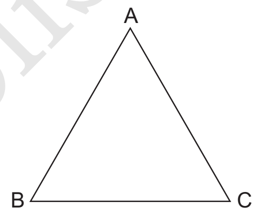
Example 2. *Calculate the measure of one interior angle of a regular hexadecagon (16 sided polygon)?*

Solution. The polygon is an hexadecagon, so we have, $n = 16$

Interior angle of a regular polygon (IA) = $\frac{(n-2)180}{n}$

18.11. TRIANGLE

A plane figure bounded by three lines in a plane is called a triangle.



Let A, B, C be three points such that all are not in a line. then, the line segments AB, BC and CA form a triangle with vertices A, B and C. The segments AB, BC and CA are called the sides and the angles BAC, ABC and ACB are called the angles of ΔABC.

Types of Triangles

Triangles are classified into various types on the basis of the lengths their sides as well as on the basis of the measure of their angles.

Following are the types of triangles on the basis of sides.

Scalene Triangle: A triangle, no two of whose sides are equal is called a scalene triangle.

Isosceles Triangle: A triangle, two of whose sides are equal is length is called an isosceles triangle.

Equilateral Triangle: A triangle, all of whose sides are equal is called an equilateral triangle.

Following are the types of triangles on the basis of angles:

Acute Triangle: A triangle, each of whose sides is acute, is called an acute triangle or an acute angled triangle.

Right Triangle: A triangle with one angle a right angle is called a right triangle or a right angled triangle.

Obtuse Triangle: A triangle with one angle an obtuse angle, is known as an obtuse triangle or obtuse angled triangle.

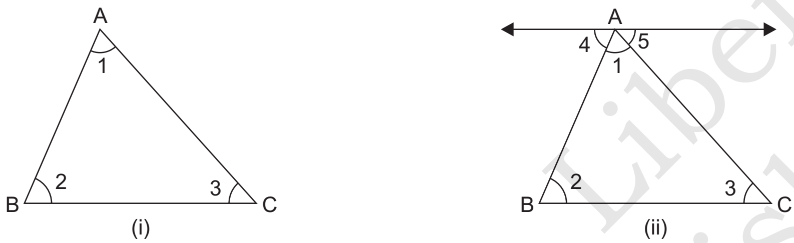
Note: It should be noted that an equilateral triangle is an isosceles triangle but the converse is not true.

18.12. ANGLE SUM PROPERTY OF A TRIANGLE

In this section, we shall deduce an important property about the tangles of a triangle, namely, that the sum of the tangles of a triangle is 180°.

Theorem 1. *The sum of three angles of a triangle is 180°.*

Given: A triangle ABC.



To Prove: $\angle A + \angle B + \angle C = 180°$ *i.e.* $\angle 1 + \angle 2 + \angle 3 = 180°$

Construction: Through A, draw a line l parallel to BC.

Proof: Since $l \parallel BC$. Therefore,

$$\angle 2 = \angle 4 \quad \text{[Alternate interior angles]}$$

and

$$\angle 3 = \angle 5 \quad \text{[Alternate interior angles]}$$

$$\therefore \quad \angle 2 + \angle 3 = \angle 4 + \angle 5$$

$$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 1 + \angle 4 + \angle 5$$

$$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = \angle 4 + \angle 1 + \angle 5$$

$$\Rightarrow \quad \angle 1 + \angle 2 + \angle 3 = 180°$$

$$\begin{bmatrix} \because & \text{Sum of angles at a point on a line is } 180° \\ \therefore & \angle 4 + \angle 1 + \angle 5 = 180° \end{bmatrix}$$

$$\Rightarrow \quad \angle A + \angle B + \angle C = 180°$$

Thus, the sum of the three angles of a triangle is 180°.

Corollary: *If the bisectors angles $\angle ABC + \angle ACB$ of a triangle ABC meet at a point O,*

then

$$\angle BOC = 90° + \frac{1}{2}\angle A$$

Given: A ΔABC such that bisectors of $\angle ABC$ and $\angle ACB$ meet at a point O.

To Prove: $\angle BOC = 90° + \frac{1}{2}\angle A$

Proof: In BOC, we have

$$\angle 1 + \angle 2 + \angle BOC = 180°$$

In ΔABC, we have

$$\angle A + \angle B + \angle C = 180°$$

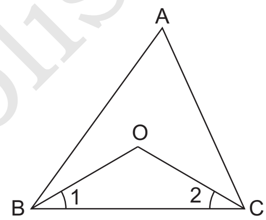
$$\Rightarrow \quad \angle A + 2(\angle 1) + 2(\angle 2) = 180°$$

$$\left[\begin{array}{ll} \because & BO \text{ and } CO \text{ are bisectors of } \angle ABC \text{ and } \angle ACB \text{ respectively} \\ \therefore & \angle B = 2\angle 1 \text{ and } \angle C = 2\angle 2 \end{array}\right]$$

$$\Rightarrow \quad \frac{\angle A}{2} + \angle 1 + \angle 2 = 90° \qquad \text{[Dividing both sides by 2]}$$

$$\Rightarrow \quad \angle 1 + \angle 2 = 90° - \frac{\angle A}{2} \qquad \ldots(ii)$$

Substituting this value of $\angle 1 + \angle 2$ in (i), we get



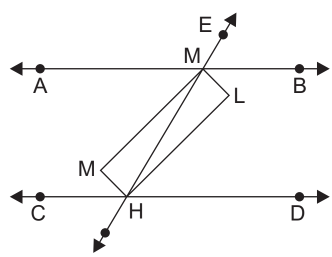
$$90° - \frac{\angle A}{2} + \angle BOC = 180°$$

$$\Rightarrow \quad \angle BOC = 180° - 90° + \frac{\angle A}{2}$$

$$\Rightarrow \quad \angle BOC = 90° + \frac{\angle A}{2}$$

Theorem 2. *If two parallel lines are interested by a transversal, prove that the bisector of the two pairs of interior angles enclose a rectangle.*

Given: Two parallel lines AB and CD and a transversl a EF intersecting them at G and H respectively. GM, HM, GL and HL are the bisectors of the two pairs of interior angles.



To Prove: GMHL is a rectangle.

Proof: We have

$$\angle AGH = \angle DHG \qquad \text{[Alternate interior angles]}$$

$$\Rightarrow \quad \frac{1}{2}\angle AGH = \frac{1}{2}\angle DHG$$

$$\Rightarrow \quad \angle HGM = \angle GHL$$

Thus, lines GM and HL are intersected by a transversal GHat G and H respectively such that pair of alternate angles are equal *i.e.*, $\angle HGM = \angle GHL$

$\because \quad GM \parallel HL$

Similarly, we can prove that $GL \parallel HM$. So, GMHL is a parallelogram.

Since $AB \parallel CD$ and EF is a transversal.

$$\therefore \quad \angle BGH + \angle DHG = 180°$$

$$\left[\because \begin{array}{l}\text{Sum of interior angles on the same}\\ \text{side of a transversal} = 180°\end{array}\right]$$

$$\Rightarrow \quad \angle LGH + \angle LGH = 90°$$

$$\left[\because \frac{1}{2}\angle BGH = \angle LGH \text{ and, } \frac{1}{2}\angle DHG = \angle LHG\right]$$

But $\angle LGH + \angle LHG + \angle GLH = 180°$

[Sum of the angles of a triangle is 180°]

$$\therefore \quad 90° + \angle GLH = 180° \qquad [\because \angle LGH + \angle LHG = 90°]$$

$$\Rightarrow \quad \angle GLH = 180° - 90°$$

$$\Rightarrow \quad \angle GLH = 90°$$

Thus, in the parallelogram GMHL, we have $\angle GLH = 90°$.

Hence, GMHL is a rectangle.

Example 3. *In ΔABC, $\angle B = 105°$, $\angle C = 50°$. Find $\angle A$.*

Solution. We have

$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad \angle A + 105° + 50° = 180°$$

$$\Rightarrow \quad \angle A = 180° - 155° = 25°$$

Example 4. *The sum of two angles of a triangle is equal to its third angle. Determine the measures of the third angle.*

Solution. Let ABC be a triangle such that

$$\angle A + \angle B = \angle C$$

We know that

$$\angle A + \angle B + \angle C = 180°$$

Putting $\angle A + \angle B = \angle C$ in (*ii*), we get

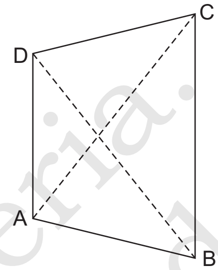
$$\angle C + \angle C = 180°$$

$$\Rightarrow \quad 2\angle C = 180°$$

$$\Rightarrow \quad \angle C = 90°$$

Thus, measure of the third angle is of 90°.

18.13. QUADRILATERAL



A four sided closed plane figure is called a quadrilateral.

Figure shows a quadrilateral ABCD in which AB, BC, CD and DA are the four sides. It is writeen as quad. ABCD or • ABCD.

The four points A, B, C and D are called its vertices.

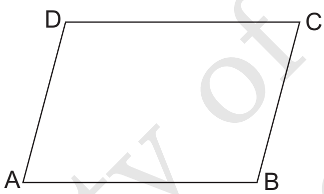
$\angle A$, $\angle B$, $\angle C$ and $\angle D$ are the four angles of quad. ABCD.

AC and BD are called the diagonals of quad. ABCD.

18.14. VARIOUS TYPES OF QUADRILATERALS

(*i*) **Parallelogram:** A quadrilateral in which both pairs of opposite sides are parallel is called parallelogram. Figure shows a parallelogram.

(*ii*) **Rectangle:** A parallelogram in which each angle is 90°, is called a rectangle. Figure shows a rectangle.

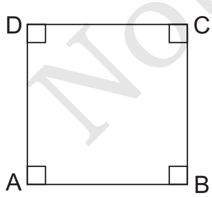


Parallelogram

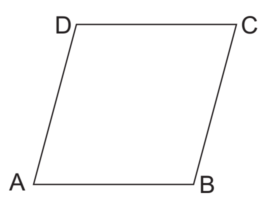
Rectangle

(*iii*) **Square:** A rectangle having all its sides equal is called a square. Figure shows a square in which AB = BC = CD = DA.

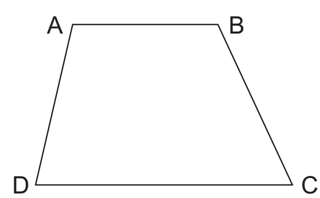
(*iv*) **Rhombus:** A parallelogram having all its sides equal is called a rhombus. Figure shows a rhombus in which AB = BC = CD = DA.



Square



Rhombus



Trapezium

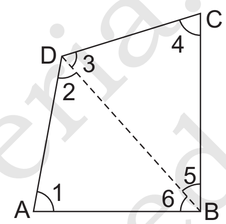
(*v*) **Trapezium:** A quadrilateral in which one pair of opposite sides is parallel is called a trapezium. Figure shows a trapezium is which AB || DC.

Theorem 3. *The sum of four angles of a quadrilateral is 360°.*

Given: ABCD is a quadrilateral

To Prove: $\angle A + \angle B + \angle C + \angle D = 360°$

Construction: Join BD



Proof: In ΔABD

$$\angle 1 + \angle 2 + \angle 6 = 180° \text{ (ASP)} \qquad \ldots(i)$$

In ΔBCD,

$$\angle 3 + \angle 4 + \angle 5 = 180° \text{ (ASP)} \qquad \ldots(ii)$$

Adding (*i*) and (*ii*), we get

$\angle 1 + \angle 2 + \angle 6 + \angle 3 + \angle 4 + \angle 5 = 180° + 180° = 360°$

$\Rightarrow \quad \angle 1 + (\angle 2 + \angle 3) + \angle 4 + (\angle 4 + \angle 6) = 360°$ (on rearranging)

$\Rightarrow \quad \angle A + \angle D + \angle C + \angle B = 360°$

$\therefore \quad \angle A + \angle B + \angle C + \angle D = 360°$

Example 5. *Three angles of a quadrilateral measure 56°, 115° and 84°. Find the measure of the fourth angle.*

Solution. Let the measure of the fourth angle be $x°$

We know sum of four angles of a quadrilateral = 360°

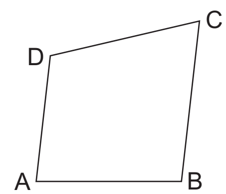
$$56° + 115° + 84° + x° = 360°$$

$$255° + x° = 360°$$

$$\therefore \quad x° = 360° - 255° = 105°.$$

Example 6. *The angles of a quadrilateral measure 2 : 4 : 5 : 7. Find the angles.*

Solution. In quadrilateral ABCD



$\Rightarrow \quad \angle A = 2x$

$\Rightarrow \quad \angle B = 2x$

$\Rightarrow \quad \angle C = 5x$

$\Rightarrow \quad \angle D = 7x$

We know

$$\angle A + \angle B + \angle C + \angle D = 360°$$

$$2x + 4x + 5x + 7x = 360°$$

$$\Rightarrow \quad 18x = 360°$$

$$\therefore \quad x = 20°$$

$\angle A = 2 \times 20° = 40°$; $\angle B = 4 \times 20° = 80°$; $\angle C = 5 \times 20° = 100°$;

$\angle D = 7 \times 20° = 140°$.

18.15. CIRCLE

A circle is a the locus of a point which moves in a plane in such a way that its distance from a given fixed point in the plane is always constant.

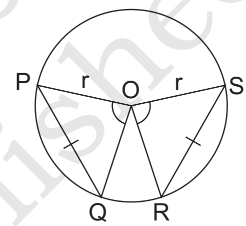
There are many circle theorem are given below:

Theorem 4. *Equal chords of a circle subtend equal angles at the centre.*

Given: A circle C(o, r) in which PQ and RS are two equal chords *i.e.*, PQ = RS

To Prove: ∠POQ = ∠ROS

Proof: In ΔPOQ and ΔROS,



OP = OS (Radii of a circle)

OQ = OR (Radii of a circle)

PQ = RS (given)

ΔPOQ ≅ ΔROS (By SSS)

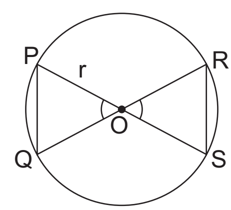
So ∠POQ = ∠ROS (By CPCT)

Theorem 5. *If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal*

Given: A circle C(o, r) in which ∠POQ = ∠ROS.

To Prove: PQ = RS

Proof: In ΔPOQ and ΔROS,



OP = OS (Radii of a circle)

OQ = OS (Radii of a circle)

∠POQ = ∠ROS (given)

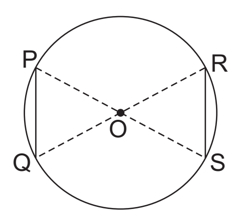
∴ ΔPOQ ≅ ΔROS (By SAS)

and so PQ = RS (By CPCT)

Theorem 6. *If two arcs of a circle are congruent, then the corresponding chords are equal.*

Given: A circle C(o, r) in which

$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$ *i.e.*, ∠POQ = ∠ROS



To Prove: PQ = RS

Construction: Join OP, OQ, OR and OS

Proof: Case I: When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are minor arcs

In $\Delta POQ = \Delta ROS$

$OP = OR$ (Radii of a circle)

$OQ = OS$ (Radii of a circle)

$\angle POQ = \angle ROS$ $(\because \overset{\frown}{PQ} \cong \overset{\frown}{RS})$

$\therefore$ $\Delta POQ \cong \Delta ROS$ (By SAS)

and so $PQ = RS$ (By CPCT)

Case II: When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are major arcs

$\overset{\frown}{QP}$ and $\overset{\frown}{SR}$ are minor arcs

$\overset{\frown}{PQ} \cong \overset{\frown}{RS}$ (given)

$\Rightarrow$ $\overset{\frown}{QP} \cong \overset{\frown}{SR}$

$\Rightarrow$ $OP = SR$

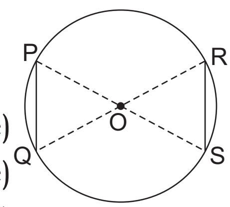
$\therefore$ $PQ = SR$

Theorem 7. *If two chords of a circle are equal then their corresponding arcs (minor, major or semicircular) are congruent.*

Given: A circle $C(o, r)$ in which

To Prove: $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$, where both $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are minor, major or semi-circular arc

Proof: Case I: When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are minor arcs



In ΔPOQ and ΔROS

$OP = OR$ (Radii of a circle)

$OQ = OS$ (Radii of a circle)

$PQ = RS$ (given)

$\therefore$ $\Delta POQ \cong \Delta ROS$ (By SAS)

and so $\angle POQ = \angle ROS$ (By CPCT)

$\therefore$ $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

Case II: When $\overset{\frown}{PQ}$ and $\overset{\frown}{RS}$ are major arcs

$\overset{\frown}{QP}$ and $\overset{\frown}{SR}$ are minor arcs

$\therefore$ $\overset{\frown}{PQ} \cong \overset{\frown}{RS}$

$\Rightarrow$ $\overset{\frown}{QP} = \overset{\frown}{SR}$

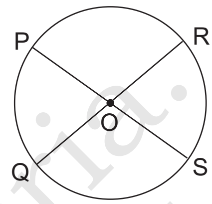
$$\Rightarrow \quad QP = SR$$

$$\Rightarrow \quad m(\widehat{QP}) = m(\widehat{SR})$$

$$\Rightarrow \quad 360° - m(\widehat{PQ}) = 360° - m(\widehat{RS})$$

$$\Rightarrow \quad m(\widehat{PQ}) = m(\widehat{RS})$$

$$\therefore \quad \widehat{PQ} \cong \widehat{RS}$$

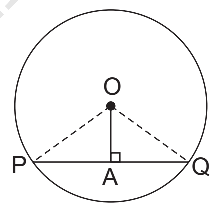


Case III: When PQ and RS are diameters. As PQ and RS are diameters, then $\widehat{PQ}$ and $\widehat{RS}$ are semicircles in which PQ = RS.

$\therefore$ They are congruent *i.e.*, $\widehat{PQ} \cong \widehat{RS}$

Theorem 8. *The perpendicular from the centre of a circle to a chord bisects the chord.*

Given: A circle C(o, r) in which PQ is a chord and QA ⊥ PQ



To Prove: PA = QA

Construction: Join OP and OQ

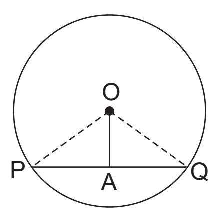
Proof: In ΔOAP and ΔOAQ

In ΔPOQ and ΔROS

	OA = OA	(Common)
	OP = OQ	(Radii of a circle)
	∠OAP = ∠OAQ	(Each 90°)
∴	ΔOAP ≅ ΔOAQ	(By RHS)
and so	PA = QA	(By CPCT)

Theorem 9. *The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.*

Given: A circle C(o, r) in which A is the midpoint of chord PQ i.e., PA = QA.



To Prove: OA ⊥ PQ

Construction: Join OP and OQ

Proof: In ΔOAP and ΔOAQ

	OP = OQ	(Radii of a circle)
	OA = OA	(Common)
	PA = QA	(Given)
∴	ΔOAP ≅ ∠OAQ	(By SSS)

and so $\angle OAP \cong \angle OAQ$ (By CPCT)

$\angle OAP + \angle OAQ = 180°$ (LPAs')

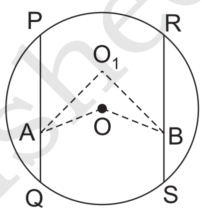
$\angle OAP + \angle OAP = 180°$

$2\angle OAP = 180°$

$\angle OAP = 90°$

Hence, $OA \perp PQ$

Corollary: *Prove that the perpendicular bisectors of two chord of a circle intersect at its centre.*



Given: A circle C(*o, r*) in which PQ and RS are two chords. Let O_1A and O_1B are perpendicular bisectors of PQ and RS respectively which intersect at O_1.

To Prove: O_1 coincides with O

Construction: Join OA and OB

Proof: A is the midpoint of PQ

($\because$ O_1A is the perpendicular bisector of PQ)

$\Rightarrow$ $OA \perp PQ$

$\Rightarrow$ OA is the perpendicular bisector of PQ

$\Rightarrow$ OA and O_1A both are perpendicular bisector of PQ

$\Rightarrow$ O_1A lies along OA

Similarly, B is the midpoint of RS.

$\Rightarrow$ $OB \perp RS$

$\Rightarrow$ OB is the perpendicular bisector of RS

$\Rightarrow$ OB and O_1B both are perpendicular bisector of RS

$\Rightarrow$ O_1B lies along OB

Thus O_1A lies along OA and O_1B lies along OB. This means that the point of intersection of O_1A and O_1B coincides with the point of intersection of OA and OB *i.e.,* O_1 coincides with O.

Hence the perpendicular bisector of PQ and RS intersect at the centre of the circle.

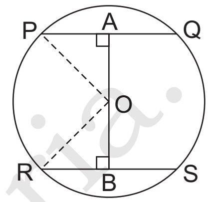
Theorem 10. *Equal chords of a circle are equidistant from the centre.*

Given: A circle C(*o, r*) in which PQ = RS

To Prove: OA = RS

Construction: Join OP and OR

Proof: We know that the perpendicular drawn from centre to a chord bisects the chord



$\therefore \quad PA = \frac{1}{2}PQ$ and $RB = \frac{1}{2}RS$

$PQ = RS$ (Given)

$\Rightarrow \quad \frac{1}{2}PQ = \frac{1}{2}RS$

$\therefore \quad PA = RB$ (Radii of a circle)

$PA = RB$ (Proved)

$\angle OAP = \angle OBR$ (Each 90°)

$\therefore \quad \angle OAP \cong \angle OBR$ (By RHS)

and so $\quad OA = OB$ (By CPCT)

Theorem 11. *Chords equidistant from the centre of a circle are equal in length.*

Given: A circle C(*o*, *r*) in which $OA \perp PQ$ and $OB \perp RS$. OA = OB.

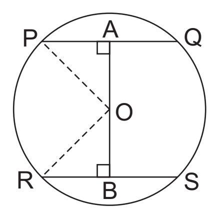
To Prove: PQ = RS

Construction: Join OP and OR

Proof: We know that the perpendicular drawn from centre to a chord bisects the chord

$\therefore \quad PA = \frac{1}{2}PQ \Rightarrow PQ = 2PA$

$RB = \frac{1}{2}RS \Rightarrow RS = 2RB$



In ΔOAP and ΔOBR

$OP = OR$ (Radii of a circle)

$OA = OB$ (Given)

$\angle OAP = \angle OBR$ (Each 90°)

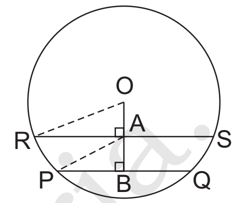
$PA = RB$

and so $\quad 2PA = 2RB$

so $\quad PQ = RS$

Example 7. *PQ and RS are two chords of a circle such that PQ = 6 cm and RS = 12 cm and PQ || RS. If the distance between OQ and RS and 3 cm, find the radius of the circle.*

Solution. PQ = 6 cm, RS = 12 cm



$$PQ \parallel RS, \ AB = 3 \text{ cm}$$

Let $\quad OP = OR = r$

and $\quad OA = x$

$$OB = OA + AB = x + 3$$

In ΔOAR $\quad OR^2 = OA^2 + RA^2$

$$r^2 = x^2 + 6^2 \qquad \left(\because RA = \frac{1}{2}RS\right)$$

$\therefore \quad r^2 = x^2 + 36$

In ΔOBP $\quad OP^2 = OB^2 + PB^2$

$$r^2 = (x+3)^2 + 3^2 \qquad \left(\because PB = \frac{1}{2}PQ\right)$$

$$x^2 + 36 = (x+3)^2 + 9 \qquad \text{[Using } (i)\text{]}$$

$$x^2 + 36 = x^2 + 6x + 9 + 9$$

$\Rightarrow \quad 36 = 6x + 18$

$\Rightarrow \quad 36 - 18 = 6x$

$\Rightarrow \quad 18 = 6x$

$\therefore \quad x = \dfrac{18}{6} = 3 \text{ cm}$

Putting the value of x in (i), we get

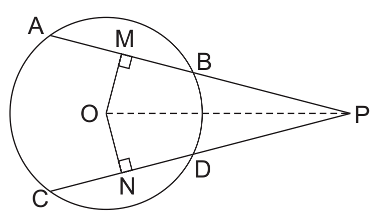
$$r^2 = 3^2 + 36$$

$$r^2 = 9 + 36 = 45$$

$$r = \sqrt{45} = 3\sqrt{5} \text{ cm}$$

Example 8. *Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.*

Solution. Given: AB and CD are two equal chords of a circle



To Prove: PB = PD

Construction: Join OP

Proof: PM = ON

($\because$ AB = CD equal chords are equidistance from centre)

In ΔOMP and ΔONP

$$\angle M = \angle N \qquad \text{(Each } 90°\text{)}$$

$$OM = ON \qquad \text{(Proved)}$$

$$OP = OP \qquad \text{(Common)}$$

$\therefore$ $$\Delta OMP \cong \Delta ONP \qquad \text{(By RHS)}$$

and so $$PM = PN \qquad \text{(By CPCT)}$$

$$AB = CD \qquad \text{(Given)}$$

$$\frac{1}{2}AB = \frac{1}{2}CD$$

($\because$ Perpendicular drawn from centre to a chord bisects the chord)

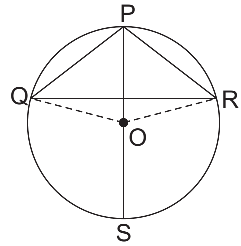
$$BM = DN \qquad \ldots(ii)$$

Subtracting (*i*) and (*ii*), we get

$$PM - BM = PN - DN$$

$\therefore$ $$PB = PD$$

Example 9. *PQ and PR are equal chords drawn on opposite sides of a diameter PS. Prove that PS is the bisector of ∠QPR.*



Solution. Given: PQ and PR are two equal chords. PS is the diameter of the circle

To Prove: $\angle OPQ = \angle OPR$

Construction: Join QR, OQ and OR

Proof: In ΔPOQ and ΔPOR

$$PQ = PR \qquad \text{(Given)}$$

$$OQ = OR \qquad \text{(Radii of a circle)}$$

$$OP = OP \qquad \text{(Common)}$$

$\therefore$ $$\Delta POQ \cong \Delta POR \qquad \text{(By SSS)}$$

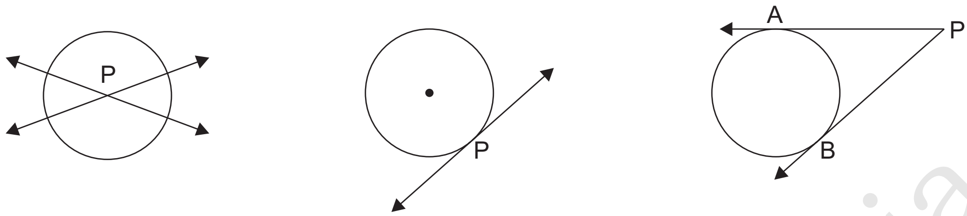
and so $$\angle OPQ = \angle OPR \qquad \text{(By CPCT)}$$

18.16. NUMBER OF TANGENTS FROM A POINT ON A CIRCLE

(*i*) If a point inside the circle it is not possible to draw any tangent to a circle through this point as shown in figure (*a*).

(*ii*) If a point is on the circle then only one tangent to the circle through this point can be drawn as shown in (*b*).

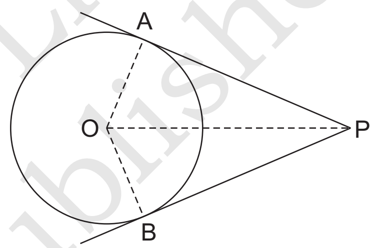
(*iii*) If a point is outside the circle then exactly two tangents can be drawn to the circle through this point as shown in (*c*).



(a) (b) (c)

Theorem 12. *The lengths of tangents drawn from an external point to a circle are equal.*

Given: A circle with center O. PA and PB are tangents drawn from external point P to the circle.



To Prove: AP = BP

Construction: Join OA, OB andOP

Proof: We known that a tangent drawn from a point to a cricle is perpendicular to the radius at the point of contact.

$\therefore$ OA $\perp$ AP and OB $\perp$ BP.

In ΔOAP and ΔOBP

	$\angle A = \angle B$	(Each 90°)
	$OA = OB$	(Radius of the circle)
	$OP = OP$	(Common)
	$\Delta OAP \cong \Delta OBP$	(By RHS)
and so	$AP = BP$	(By CPCT)

Theorem 13. *If two tangents are drawn from an external point to a circle then:*

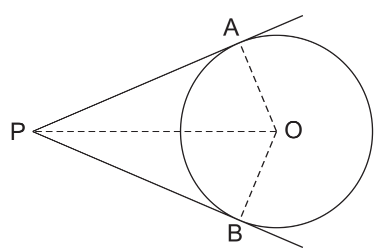
(*i*) *they subtend equal angles at the centre*

(*ii*) *they are equally inclined to the line joining the centre to the point.*

Given: A circle with center O. PA and PB are tangents drawn from point P to the circle.

To Prove: (*i*) $\angle AOP = \angle BOP$

(*ii*) $\angle APO = \angle BPO$



Construction: Join OA, OB andOP

Proof: We know that a tangent drawn from a point to a circle is perpendicular to the radius at the point of contact.

$\therefore$ OA $\perp$ AP and OB $\perp$ BP.

In ΔOAP and ΔOBP

$$OA = OB \qquad \text{(Radius of a circle)}$$

$$AP = BP$$

$$(\because \text{ Tangents from an external point are equal})$$

$$OP = OP \qquad \text{(Common)}$$

$\therefore$ $\Delta OAP = \Delta OBP$ (By SSS)

and so $\angle AOP = \angle BOP$ (By CPCT)

$\angle APO = \angle BPO$ (By CPCT)

Example 10. *In a ΔABC, if $2\angle A = 3\angle B = 6\angle C$, determine $\angle A$, $\angle B$ and $\angle C$.*

Solution. We have,

$$2\angle A = 3\angle B = 6\angle C$$

$$\frac{\angle A}{3} = \frac{\angle B}{2} = \frac{\angle C}{1}$$

[Dividing throughout by 6 *i.e.* by the L.C.M of 2, 3 and 6]

$\Rightarrow$ $\angle A : \angle B : \angle C = 3 : 2 : 1$

$\Rightarrow$ $\angle A = 3x$, $\angle B = 2x$ and $\angle C = x$. Then,

$\Rightarrow$ $\angle A + \angle B + \angle C = 180°$

$\Rightarrow$ $3x + 2x + x = 180°$

$\Rightarrow$ $6x = 180°$

$\Rightarrow$ $x = 30°$

Hence, measures of the angles of the triangle are

$\Rightarrow$ $\angle A = 3x = 90°$

$\Rightarrow$ $\angle B = 2x = 60°$

$\Rightarrow$ $\angle C = x = 30°$

Example 11. *A triangle ABC is right angled at A. AL is drawn perpendicular to BC. Prove that $\angle BAL = \angle ACB$.*

Solution. In AB ΔABL, we have

$\angle BAL + \angle ALB + \angle B = 180°$

$\Rightarrow$ $\angle BAL + 90° + \angle B = 180°$ $[\because$ AL $\perp$ BC $\therefore$ $\angle ALB = 90°]$

$\Rightarrow$ $\angle BAL + \angle B = 90°$

$\Rightarrow$ $\angle BAL = 90° - \angle B$ ...(*i*)

In ΔABC, we have

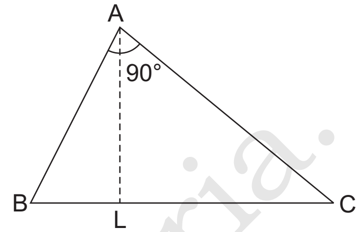
$$\angle A + \angle B + \angle C = 180°$$

$$\Rightarrow \quad 90° + \angle B + \angle C = 180°$$

$$\Rightarrow \quad \angle B + \angle C = 180° - 90°$$

$$\Rightarrow \quad \angle B + \angle C = 90°$$

$$\Rightarrow \quad \angle C = 90° - \angle B$$



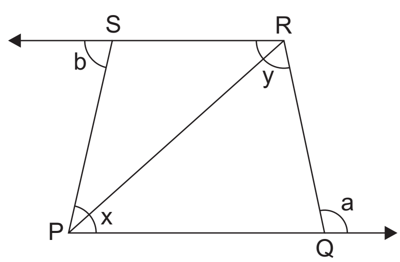
$$\Rightarrow \quad \angle ACB = 90° - \angle B \qquad \ldots(ii)$$

From (*i*) and (*ii*), we get

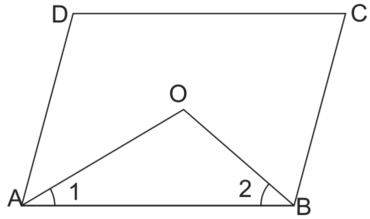
$$\angle BAL = \angle ACB$$

EXERCISE

1. Calculate the measures of 1 exterior angle of a regular pentagon?
2. Find the sum of all the interior angle of a polygon having 29 sides.
3. Of the three angles of a triangle, one is twice the smallest and another is three times the smallest. Find the angles.
4. If the angles of a triangle are in the ratio 2 : 3 : 4, determine three angles.
5. The sum of two angles of a triangle is 80° and their difference is 20°. Find all the angles.
6. Find the length of tangent drawn to a circle with radius 5 cm from a point 13 cm away from the centre of the circle.
7. If the radii of the two concentric circles are 15 cm and 17 cm, show that the length of the chord of one circle which is tangent to other circle is 16 cm.
8. If the sum of the measure of the interior angle of polygon is 3240, find the number of sides of the polygon.
9. Find the sum of interior angles of a decagon.
10. Sum of all interior angles of a polygon is 3060°. How many sides does the polygon have?
11. The sides PQ and RS of a quadrilateral PQRS are produced as shown in figure. Prove that $a + b = x + y$.



12. In a parallelogram, show that the angle bisectors of two adjacent angles intersect at right angles.



13. ABCD is a rectangle with $\angle BAC = 42°$. Determine $\angle DBC$.